is also made with $X = 2.5$ (T or Q) $- 1.5R$, and the best (with the least F value) of T or Q or X is used for replacement in A. The iteration process is continued until the F_R value assumes the prefixed threshold value.

Illustrative Results

A computer code based on the foregoing methodology was developed in Fortran IV on a Control Data Corporation CYBER 170/730 computer. Optimum flight times and the profiles for θ for transfer from Earth to Mercury, Venus, and Martian orbits, in the ideal case when both the terminal orbits are regarded as circular and coplanar, are obtained and are found to be in good agreement with those reported in Refs. 1 and 2. For a typical illustration, details of the optimal Earth-Mercury transfer with the considerations of the ellipticity of both the terminal orbits are presented in Table 1 for $\alpha = 2$ mm/s^2. As may be inferred from Table 1, for any departure point on the Earth's orbit there is a corresponding true anomaly at arrival for rendezvous, and there are two extrema for T_F termed opt-min and opt-max occurring when ν_1 is nearly 0 and 150 deg. The maximum variation in T_F is ~ 12 days. Graphical representations for e and θ are attempted in Fig. 1. However, it may be pointed out that, for smaller accelerations, rendezvous becomes difficult and even impossible for some regions of departure. For example, when $\alpha = 1$ mm/s^2, no rendezvous is found to be possible when ν_1 is in the region of ~ 300–145 deg. Details relating to the convergence of the optimization algorithm are given in Ref. 8.

Conclusions

A convenient formulation for determining the effect of the eccentricities of the terminal orbits on the steering profile of a sail spacecraft for time-optimal rendezvous transfer between coplanar heliocentric orbits is presented. The optimal control strategy is attempted by conversion to a two-point boundary-value problem for a system of seven ordinary differential equations. Effectiveness of the CRS optimization technique for the solution is demonstrated.

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Significance of Modeling Internal Damping in the Control of Structures

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Introduction

THIS Note examines the significance and importance of modeling the internal damping of structures in designing feedback control laws for vibration suppression. The authors have in mind control problems for large flexible spacecraft. In the description of control of flexible space structure, the models used are large-order finite element models or partial differential equation models. However, the point of this Note can be made by examining simple one and two degree-of-freedom models as well as simple hybrid-distributed-parameter models. Damping in spacecraft is often ignored in control design because it is difficult to model and measure. Here several simple systems are examined to illustrate that control design with ignorance of damping has the potential for resulting in poor performance or even an unstable closed-loop response.

The negative effects of unmodeled damping are pronounced in systems that do not use collocated sensors and actuators. However, as shown in Ref. 1, better performance is obtained by noncollocated feedback laws. It has been shown that if the actuator dynamics are significant compared to those of the structure (often the case), then collocated sensors and actuators are not possible (see Ref. 1). Hence, a noncollocated feedback law is also considered here.

We also consider an example [the Rocket Propulsion Laboratory (RPL) structure] for which even the actuators (a tip jet nozzle and flexible hose) for a simple beam produce significant damping, which, if ignored in the basic system modeling, results in a model that cannot yield a reasonable time response using physically meaningful parameter values. Such a model also yields a less than satisfactory result when used in a control design.

Poorly Estimated Damping in Lumped-Parameter Systems

It is instructive to first consider simple velocity feedback control of a single degree-of-freedom oscillator. In this case, it is easy to verify that the estimate of the value of the open-loop damping coefficient greatly effects the closed-loop response. If the open-loop damping coefficient is underestimated, then the feedback control gain will be chosen to be smaller than actually needed and the desired closed-loop response will not be obtained. If, on the other hand, the open-loop damping coefficient is overestimated, a velocity feedback law with the objective of decreasing the system's speed of response (such as in motor control) can actually produce an unstable system (see Ref. 2).

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This potential for instabilities to result in the closed-loop response through the inadvertent reduction in gain margin because of poorly modeled damping increases for multiple degree-of-freedom systems. Consider a multiple degree-of-freedom model for a structure consisting of a series connection of n spring-mass damper arrangements. Attached to this model are a number of proof-mass actuators (see Refs. 3 and 4) modeled as simple oscillators which account for their dynamics. The closed-loop stability of such actuator/structure systems is discussed in Ref. 1 for the case of velocity feedback. The results obtained in Ref. 1 can be used to discuss the effects of poor damping measurements on closed-loop stability.

For an n degree-of-freedom system with m reaction mass actuators, the equation of motion can be written in closed-loop form as

$$M\ddot{y} + (D + G)\dot{y} + Ky = 0 \tag{1}$$

where M, D, and K are real positive definite symmetric mass, damping, and stiffness matrices, respectively, and y is an $(m + n) \times 1$ vector of coordinates. The matrix G represents a matrix of control gains calculated for velocity feedback. It is shown in Ref. 1 that closed-loop stability is guaranteed if

$$4(d_i + d_{i+1})d_{ia} > g_i^2, \qquad i = 1, 2, \ldots, m \tag{2}$$

where the subscript i denotes the ith actuator, g_i is the gain of the ith actuator, d_{ia} is internal damping of the ith actuator, and d_i and d_{i+1} are the damping coefficients of the structure at the location of the ith actuator. Equation (2) represents a constraint on the choice of the control gains g_i and its value depends critically on valid measurements of d_i and d_{i+1}, the internal damping of the structure. As in the single degree-of-freedom case, if the internal damping parameters are underestimated, then Eq. (2) indicates that g_i should be limited to a lower value than need be. Hence, the performance of the closed-loop system will be less than is possible, i.e., the closed-loop control will not achieve its potential. If, on the other hand, d_i is overestimated, the closed-loop response could become unstable since the bound on g_i implied by inequality (2) could be violated for that index. Case 6 of the four degree-of-freedom, two-actuator velocity feedback examples of Ref. 1 demonstrates a situation for which this instability does occur. Of course other types of control are available. To investigate the effect of poorly measured damping on closed-loop stability is really a question of robustness (see Ref. 5). Both performance robustness and stability robustness address the question of control in the presence of uncertainty. For the case of stability robustness, some criteria are available that relate uncertainties in the closed-loop system parameters to stability.

For example, in Ref. 5 it is shown that if A is the closed-loop state matrix and ϵ is the largest additive uncertainty in A, then stability can be guaranteed by calculating a bound on the uncertainty. Let F denote the matrix solution to the Lyapunov equation

$$A^T F + F A^T = -2I \tag{3}$$

where I is the identity matrix. Let $\sigma_{\max}[F]$ denote the largest singular value of the matrix F. Then the closed-loop system is asymptotically stable if

$$\epsilon < (1/2n)(1/\sigma_{\max}[F]) \tag{4}$$

where n is the number of degrees of freedom of the structural model.

As an example of the use of the inequality in Eq. (4) to determine the effect of a poorly estimated damping parameter on closed-loop stability, consider the system Eq. (1). In first-order, vector state-space form, the closed-loop state matrix is

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}(D + G) \end{bmatrix} \tag{5}$$

To calculate the robustness bound of Eq. (4), numerical values of M, D, K, and G are needed. Using $M = I$,

$$D = \begin{bmatrix} 4 & -1 \\ -1 & 1 \end{bmatrix}, \qquad K = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$$

and a collocated symmetric velocity feedback gain matrix of the form

$$G = \begin{bmatrix} 5 & -3 \\ -3 & 3 \end{bmatrix}$$

we obtain a heavily damped closed-loop system. The corresponding closed-loop state matrix is

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 1 & -9 & 4 \\ 1 & -1 & 4 & -4 \end{bmatrix}$$

The solution of Eq. (3) yields the matrix F given by

$$F = \begin{bmatrix} 3.5495 & -0.6593 & 0.3626 & 0.0879 \\ 0.6593 & 4.6593 & 0.6374 & 1.6374 \\ 0.3626 & 0.6374 & 0.3956 & 0.5495 \\ 0.0879 & 1.6374 & 0.5495 & 1.2088 \end{bmatrix}$$

The largest value of F is computed to be $\sigma_{\max} = 5.578$ and from Eq. (4), one desires

$$\epsilon = 1/4\,(1/5.578) = 0.0448 \tag{6}$$

Hence if the measured damping elements are off by more than 0.0448 (e.g., more than about 1% relative error), stability can no longer be guaranteed. Since standard modal testing methods often predict damping that is as much as 50% off (see for instance Ref. 6), there is a clear need for improved damping estimates and measurements in structures that are to be controlled (or alternatively a need for a different control scheme).

Currently, robust control design methods have centered around a technique called "H_∞" design which takes its name from the norm of the same notation. Basically these methods generalize the use of classical Bode plots to the design of multi-input/multi-output control systems by looking at the singular values of the frequency-response function matrix. The procedure is to express stability regions directly on "Bode plots" of these singular values. This process is referred to as singular-value loop shaping. In the case of a single degree-of-freedom system, these singular-value Bode plots reduce to the usual Bode magnitude plot. It is well known that the damping parameter has a substantial impact on the magnitude plot in the single degree-of-freedom case. Hence, it is a simple matter to see how an incorrectly measured damping coefficient can spoil the closed-loop stability.

Poorly Estimated Damping in Distributed-Parameter Systems

The preceding remarks on potential difficulties related to poor quantitative estimates of damping parameters in lumped-parameter systems are also of valid concern in control of distributed-parameter systems (DPS). This may be illustrated with a brief consideration of the typical finite element method (FEM) approach (including modal techniques) that is standard in the engineering literature on modeling of DPS. The FEM approach yields an n-dimensional model of the form

$$M\ddot{y} + Ky = f \tag{7}$$

for the nodal coordinates y if general finite elements are used (or modal coordinates y if modes are used). Damping is added through an assumed damping term of the form $D\dot{y}$ with the most common assumption in general FEM models being $D = \alpha M + \beta K$, where the scalars α and β must be chosen. In the case of modal coordinates, the matrix D is most often a diagonal matrix made up of modal damping parameters. In either case, this leads to an nth degree-of-freedom model which, with velocity feedback, is of the form of Eq. (1), if the actuators have also been included in the modeling.

However, accurate modeling of damping in DPS offers additional challenges. For example, it has been shown in Ref.7 that in composite material DPS the damping does not decouple across the modes and hence damping is not modal in nature. Thus, the modal coordinate diagonal damping matrix approach alluded to in the preceding paragraph to arrive at models of the form of Eq. (1) is often inappropriate. More fundamentally, the form (as well as quantitative estimates for related parameters) of the damping may be a significant issue. Questions range from the type of physical hypothesis (e.g., strain-rate, bending-rate, time hysteresis, spatial hysteresis—see Refs. 7–10) one should postulate as a basis for the damping law, to the most appropriate way one might model a structural actuator which itself contains significant internal damping in a nontrivial form. This latter question is exemplified by the so-called RPL structure (see Refs. 11 and 12) which we shall discuss here briefly to illustrate some specific difficulties related to damping assumptions.

The RPL structure, which is depicted in Fig. 1, was designed and constructed at the Charles Stark Draper Laboratory with funding provided by the U.S. Air Force Rocket Propulsion Laboratory (RPL). This structure was developed to serve as a test bed for the implementation and validation of control algorithms for large-angle slewing maneuvers of spacecraft with flexible appendages. A more complete description of the structure and its control actuators can be found in Refs. 11 and 12. Of primary interest to us here are the flexible appendages, two of which are the so-called active appendages. These consist of flexible beams fixed to the hub at one end with nitrogen gas thrusters attached at the free ends as shown in the insert of Fig. 1. Nitrogen gas from tanks mounted on the central hub is supplied to the thrusters via stainless-steel mesh wrapped high-pressure hoses. Thrust levels are controlled by electromechanical (solenoidal) valves.

There are several ways to model the cantilevered beam with tip mass (the thruster unit) and flexible hose attached to the free end as pictured in the insert of Fig. 1. Here we summarize briefly the detailed findings of Ref. 12. The most straightforward approximation is to neglect the hose effects and any internal damping, and to treat the structure as a simple undamped cantilevered Euler-Bernoulli beam with tip mass m_T at the free end. If the fixed end is taken at $x = 0$ and the free end at $x = l$, the transverse displacement $y(t, x)$ at time t at position x along the beam is described by the well-known partial differential equation,

$$\rho \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2}\right) = 0, \qquad 0 < x < l, \qquad t > 0 \tag{8}$$

with boundary conditions

$$y(t, 0) = \frac{\partial y}{\partial x}(t, 0) = 0, \qquad t > 0 \tag{9}$$

$$\frac{\partial}{\partial x}\left(EI \frac{\partial y}{\partial x}\right)\Big|_{x=l} = m_T \frac{\partial^2 y}{\partial t^2}(t, l), \qquad t > 0 \tag{10}$$

$$EI \frac{\partial^2 y}{\partial x^2}(t, l) = 0, \qquad t > 0 \tag{11}$$

If this model is used with physical values for the parameters ρ, E, I, and m_T, one obtains a response that is incorrect when compared with experimental data for the structure—the model response is out of phase and the model frequencies are not reasonable approximations to the experimentally observed frequencies. A finite element approximation to Eqs. (8–11) with modal damping added in the usual engineering approach offers little promise toward arriving at a physically realistic model.

One way to circumvent the difficulties with Eqs. (8–11) is to attempt to account for damping (both that internal to the beam and any that might be associated with the hose/thruster assembly) by changes in values for m_T and E. In Ref. 11, Floyd employs "model adjustment" wherein m_T and E are adjusted using data in the frequency domain so that the model matches the first three modes (the low-frequency modes). However, if these new (nonphysical) values are used in the model, Eqs. (8–11), the time response still does not match the experimental time histories; it is badly out of phase (by almost 180 deg) with the data as well as underdamped. Further adjustment of the values of m_T and E is possible by using an output least-squares formulation to compare the model response with time-history data and optimizing over values of m_T and E. The resulting response offers a much better fit to the data but the resulting parameter values for m_T and E are still significantly nonphysical. In either case ("model adjustment" or output least-squares data matching), the nonphysical parameter values are clearly dependent on a specific experimental response and hence lead to a model that is inadequate for control law design.

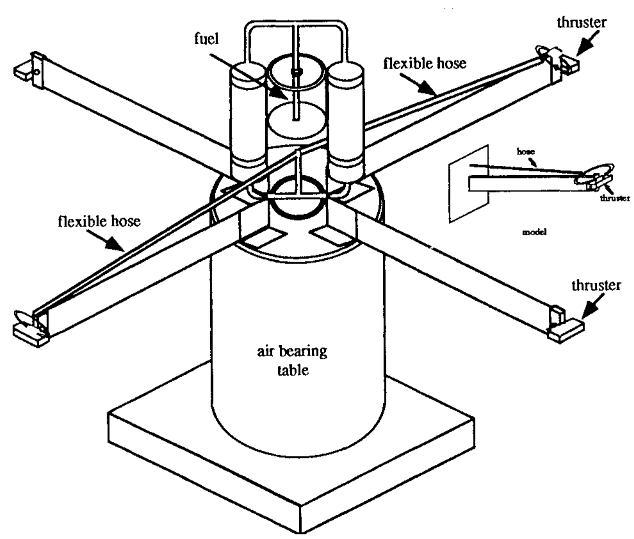

Fig. 1 A schematic of the U.S. Air Force Rocket Propulsion Lab's slewing flexible spacecraft experiment.

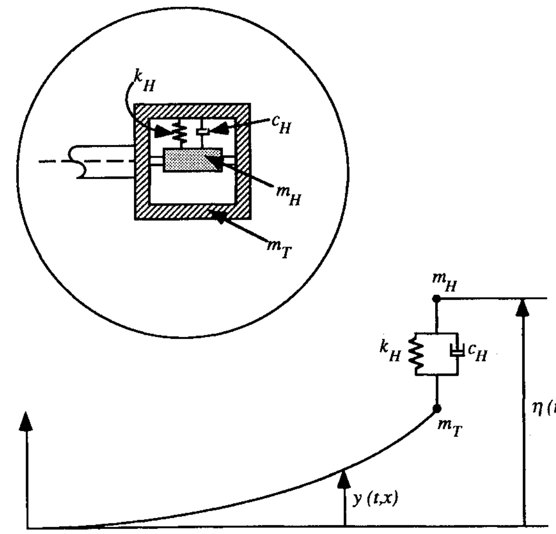

Fig. 2 A model of the active appendage of the RPL experiment of Fig. 1.

The basic tacit assumption in the just mentioned approach is that the hose/thruster assembly and the internal beam structural damping simply add mass to the tip mass m_T and increase stiffness E of the beam. This is, in essence, assuming an incorrect form for the damping.

A better model of the active appendage involves directly hose/thruster assembly effects as well as internal beam damping. A reasonable model includes the hose/thruster assembly as a point mass (dead mass) m_T at the tip end plus an attached hose assembly containing mass, stiffness, and damping as depicted in Fig. 2. A good model for the hose assembly then might be a mass/spring/dashpot system containing hose stiffness (spring constant) k_H, hose damping c_H, and hose assembly mass m_H.

Letting $\eta(t)$ represent displacement of the hose assembly center of mass from the equilibrium position of the tip end of the beam so that $\eta(t) - y(t, l)$ is the displacement of the hose assembly center of mass from the tip end of the beam, we find the following system of equations and boundary conditions for the configuration of Fig. 2:

$$\rho \frac{\partial^2 y}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(EI \frac{\partial^2 y}{\partial x^2} + c_D I \frac{\partial^3 y}{\partial x^2 \partial t}\right) = 0, \qquad 0 < x < l \tag{12}$$

$$y(t, 0) = \frac{\partial y}{\partial x}(t, 0) = 0 \tag{13}$$

$$m_H \frac{\mathrm{d}^2 \eta}{\mathrm{d}t^2}(t) + c_H\left[\frac{\mathrm{d}\eta}{\mathrm{d}t}(t) - \frac{\partial y}{\partial t}(t, l)\right] + k_H[\eta(t) - y(t,l)] = 0 \tag{14}$$

$$m_T \frac{\partial^2 y}{\partial t^2}(t, l) - \frac{\partial}{\partial x}\left\{EI \frac{\partial^2 y}{\partial x^2} + c_D I \frac{\partial^3 y}{\partial x^2 \partial t}\right\}\Bigg|_{x=1} = c_H\left(\frac{\partial \eta}{\partial t}(t) - \frac{\partial y}{\partial t}(t, l)\right) + k_H[\eta(t) - y(t, l)] \tag{15}$$

$$EI \frac{\partial^2 y}{\partial x^2}(t, l) + c_D I \frac{\partial^3 y}{\partial x^2 \partial t}(t, l) = 0 \tag{16}$$

with each expression holding for $t > 0$. Here $c_D I$ represents the beam internal damping coefficient where a Kelvin-Voigt type of internal damping (i.e., strain-rate damping) is assumed. Fixed, physically measurable values of m_T, EI, ρ, and estimates of the Kelvin-Voigt damping coefficient c_D and the hose assembly parameters m_H, c_H, and d_H employing an output least-squares formulation can be obtained. These parameters are physically reasonable. When they are used in the model, Eqs. (12-16), an excellent fit of the model time response to the experimental data is obtained.

Conclusion

These discussions are intended to illustrate the importance of the estimation of damping parameters in closed-loop performance and stability. A poorly identified damping coefficient can cause a seemingly stable closed-loop system to become unstable. Although robust control methods are able to overcome the problem to some degree, it is still possible to design an unstable closed-loop system if the damping parameter is overestimated. Even more fundamental issues as to the form of damping in structures and actuators are also important in distributed-parameter systems. (That damping form plays a fundamental role in control design has also been noted in Ref. 13.) One must conclude that correctly modeling damping form and accurately estimating damping parameters are extremely important if not essential if one's goal is to control the structure under study.

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